Probability in High Dimension and Applications

Course Info Sheet

Dept. of EE-systems
Tel-Aviv University
Fall 2020-2021

Lecturer:	Prof. Ofer Shayevitz
+ Office:	EE labs bldg., room #127
+ Email:	ofersha@tauex.tau.ac.il
+ Office hours:	Please coordinate by email

Lectures Online, asynchronous.

Syllabus

This course provides an introduction to modern techniques in the analysis of random structure in high dimension, with an emphasis on applications in information theory, communications, statistics, random matrix theory, combinatorics, and learning. The course will cover a subset of the following topics, time permitting:

- Introduction and basic inequalities (Markov, Chebyshev, Chernoff).
- The concentration-of measure phenomenon.
- Variance bounds and the Efron-Stein inequality.
- Introduction to Markov semigroup theory.
- Poincare (spectral gap) inequalities (PI):
  - PI and exponential ergodicity.
  - Tensorization of PI.
  - Orenstein-Uhlenbeck semigroup and the Gaussian PI.
  - Sturm-Liouville semigroups and PI on an interval.
- Basic Subgaussian concentration: Hoeffding lemma, McDiarmid's inequality.
- Logarithmic Sobolev inequalities (LSI):
  - The entropy method.
  - Tensorization of entropy and LSI.
  - Gaussian LSI.
  - Connections to isoperimetry and hypercontractivity.
- Connections to strong data processing inequalities.
- The Transportation method: Marton's inequality, Talagrand's inequality.
- Influence and threshold phenomena.
- Suprema of random processes.

Course Format

This course is given completely online, and will operate asynchronously via Moodle. Complete lectures and homework material will be regularly posted, and remain accessible throughout the semester. An online forum will be available for Q&A, and discussions on the forum are encouraged. All announcements posted on Moodle are binding.

Homework

Homework submission is mandatory. Each student will meet (online) with the lecturer at least once during the semester, to discuss the class material and homework.

Final Project

As a final project, you will review a research paper on a topic related to the course (from a given list, of otherwise subject to the lecturer's approval). Your review should include a clear explanation of the problem addressed in the paper, an overview of the relevant background, and a clear statement of the paper's results. This will be followed by a description of the main techniques used in the proofs, and an outline of the derivation of the main claims(s). Finally, you are requested to provide a critical review of the paper (e.g., weaknesses, gaps, what can be improved, ideas for extensions, connections to other problems, etc.).

Final Grade

75% of the final grade will be determined by the project, and 25% by the homework + online meeting.

Main Literature

[1] *Probability in High Dimension*, R. Van Handel, Lecture Notes, Princeton, 2016.

[2] *Analysis and Geometry of Markov Diffusion Processes*, D. Bakry, I. Gentil and M. Ledoux, Vol. 348, Springer Science & Business Media, 2013.

Additional Literature

[3] *Concentration Inequalities*, S. Boucheron, G. Lugasi & P. Masart, Oxford University Press, 2013.

[4] *Concentration of Measure Inequalities in Information Theory, Communications and Coding*, M. Raginsky and I. Sason. Foundations and Trends in Communications and Info. Theory, 2014.

[5] *The Concentration of Measure Phenomenon*, M. Ledoux, American Mathematical Soc., 2005.